Derivatives and integrals in calculus are like opposite sides of a coin. They're fundamentally connected, and understanding this relationship is key to mastering calculus. One fundamental theorem states that if you know the indefinite integral (F) of a function f(x), you can easily calculate the definite integral (the area under the curve) between points a and b by finding the difference in the values of F at those two points: ∫abf(x)dx=F(b)-F(a). Let's break this down further. The first fundamental theorem states that for any continuous function f(x), if you take its antiderivative (integral) and then find its derivative, you'll get back to the original function f(x): F'(x)=f(x). This means that when you integrate a function with respect to its upper limit, the result is the function itself. To illustrate this, consider the example of finding the area under the curve of 2x from x=0 to x=a. The antiderivative (integral) of 2x is x^2 - a^2, and taking its derivative gives us back 2x. Another example is calculating the distance traveled by a car at constant speed: if it travels 50 km/h for one hour, the integral of this speed over that time is simply 50 km (ignoring any initial distance already covered). The second fundamental theorem shows that if you have an antiderivative F(x) of a function f(x), then the definite integral ∫abf(x)dx equals the difference in the values of F at b and a: F(b)-F(a). This makes calculating definite integrals much easier, especially when you can find their corresponding antiderivatives. For instance, if water flows into a tank at a rate given by f(t), and F(t) measures the total volume of water in the tank at time t, then the amount of water added between times a and b is simply F(b)-F(a). This principle has far-reaching applications across various fields, including astronomy (calculating planetary orbits), finance (marginal costs and profits), engineering (bending strength and motion), and more. The definite integral can be evaluated using the Fundamental Theorem of Calculus, Part 2, which states that if we find an antiderivative of the integrand, we can evaluate the integral by subtracting the values of the antiderivative at the upper and lower limits. This is denoted as F(b) - F(a), where F(x) is the antiderivative. To do this, we can divide the interval [a, b] into smaller subintervals and sum up the differences in the antiderivative over each subinterval. As the number of subintervals increases, this sum approaches the definite integral. Using the Mean Value Theorem, we can find a point in each subinterval where the derivative of the antiderivative is equal to the function value times the width of the subinterval. Substituting these values into the sum, we get F(b) - F(a) = ∑f(ci)Δx. Taking the limit as the number of subintervals approaches infinity, we get F(b) - F(a) = ∫ab f(x) dx, which is the definite integral. This theorem can be applied to evaluate definite integrals. For example, to evaluate ∫-2 2 (t^2 - 4) dt, we first find the antiderivative using the power rule for antiderivatives, and then apply the Fundamental Theorem of Calculus, Part 2. Note that when evaluating a definite integral, any antiderivative works, and the constant term cancels out. Also, the definite integral can produce a negative value, which represents a net signed area. The problem asks us to calculate the definite integral $\int_1^9$(x^(1/2) - x^(-1/2)) dx using the power rule and then apply this result to determine who will win a race between James and Kathy, two roller skaters. The integral can be broken down into simpler integrals, which can be evaluated using the power rule. The results of these integrals are: $\int_0^5$(5 + 2t) dt = 50 ft, indicating that James has skated 50 ft after 5 seconds. For Kathy, we need to integrate 10 + cos(π^2t) over the interval [0, 5]. Since sintsint is an antiderivative of cost,cost, it's reasonable to expect that an antiderivative of cos(π2t)cos(π2t) involves sin(π2t).sin(π2t). When integrating 10 + cos(π^2t), we can break down the integral into two simpler integrals: $\int_0^5$ 10 dt and $\int_0^5$ cos(π^2t)dt. The first one evaluates to 50, while the second one evaluates to approximately 50.6 ft after 5 seconds. Kathy wins by approximately 0.6 ft. Given article text here Julie executes her jumps from an altitude of 12,500 ft and starts falling at a velocity given by v(t) = 32t after exiting the aircraft. She continues to accelerate until she reaches terminal velocity. The time taken for Julie to reach terminal velocity can be found by setting up an expression involving one or more integrals representing the distance Julie falls after 30 sec. If Julie pulls her ripcord at an altitude of 3000 ft, she spends 5 seconds in free fall before her parachute opens and 400 additional feet are covered. The total time spent in the air can be calculated by adding the time taken to reach terminal velocity, the duration of free fall, the descent after canopy deployment, and the recovery time until touchdown. A function is said to be continuous at a point if its graph can be traced without any break or discontinuity at that point. In other words, there should be no jump or gap in the graph of the function at the given point. For instance, consider a function f(x)f(x) with a hole at x=a.x=a. The condition i.f(a)f(a) is defined.i.f(a)f(a) is defined ensures that the value of the function is well-defined at this point. However, it is insufficient to guarantee continuity, as shown in Figure 2.33. To determine if f(x)f(x) is continuous at a, we need to check two additional conditions: ii.limx→af(x)exists.ii.limx→af(x)exists and iii.limx→af(x)=f(a).iii.limx→af(x)=f(a). If these conditions are met, the function f(x)f(x) is said to be continuous at x=a.x=a. To check if a function is continuous at a point, we need to verify three conditions: (1) the function must be defined at that point, (2) the limit of the function as x approaches that point must exist, and (3) the limit must equal the value of the function at that point. Let's apply this definition to some examples. First, consider the function f(x) = (x^2 - 4)/(x-2). If we try to evaluate f(2), we get 0/0, which is undefined. Therefore, this function is discontinuous at x=2. Next, let's look at the function f(x) = {-(x^2)+4 if x ≤ 3; 4(x-3) if x > 3}. To determine its continuity at x=3, we need to check if the limit of f(x) as x approaches 3 exists. We find that lim x→3- f(x) = -5 and lim x→3+ f(x) = 4, so the limit does not exist. Therefore, this function is also discontinuous at x=3. Now let's consider the function f(x) = {sin(x)/x if x ≠ 0; 1 if x = 0}. We can see that f(0) = 1 and lim x→0 sin(x)/x = 1. Therefore, this function is continuous at x=0 because all three conditions for continuity are met. We also have a theorem stating that polynomials and rational functions are continuous at every point in their domains. This means that as long as the denominator of a rational function is not zero, the function will be continuous everywhere except possibly at that point. Finally, let's apply this definition to determine where the function f(x) = (x+1)/(x-5) is continuous. Using Continuity of Polynomials and Rational Functions, we know that this function will be continuous wherever its denominator is not zero, which means it will be continuous everywhere except possibly at x=5. The given rational function f(x) = x/(x-5) has a single discontinuity at x = 5. Another rational function, g(x) = 3x^4 - 4x^2, is continuous for all values of x except where? To classify discontinuities, we consider three types: removable, jump, and infinite. A removable discontinuity occurs when there's a hole in the graph, a jump discontinuity happens when the sections of the function don't meet up, and an infinite discontinuity appears at a vertical asymptote. The article then defines these terms formally: * Removable discontinuity: if lim(x→a) f(x) exists * Jump discontinuity: if lim(x→a-) f(x) and lim(x→a+) f(x) both exist but are not equal * Infinite discontinuity: if either lim(x→a-) f(x) or lim(x→a+) f(x) is ±∞ The article then applies these definitions to two specific examples: 1. A function f(x) = x^2 - 4x - 2 has a removable discontinuity at x = 2, because the limit as x approaches 2 exists and equals 4. 2. A function f(x) = { -x^2 + 4 if x ≤ 3, 4x - 8 if x > 3 } has a jump discontinuity at x = 3, because the one-sided limits exist but are not equal. Finally, the article discusses whether another function f(x) = x + 2/x + 1 is continuous at x = -1. It finds that the function is not continuous at this point and determines that it has an infinite discontinuity there, since the limit as x approaches -1- is -∞ and the limit as x approaches -1+ is ∞. The function f(x) has an infinite discontinuity at x = -1. To determine if f(x) is continuous at x = 1, we examine its behavior on either side of this point. If it is not continuous, we need to classify the type of discontinuity (removable, jump, or infinite). Continuity can be explored in different intervals, and a function is considered continuous over an interval if we can draw the graph between any two points without lifting our pencil. To define continuity on an interval, we look at what it means for a function to be continuous from the right and left at a point. A function f(x) is said to be continuous from the right at x = a if limx→a+ f(x) = f(a), and from the left at x = a if limx→a- f(x) = f(a). A function is continuous over an open interval if it's continuous at every point, while a function is continuous over a closed interval of the form [a,b] if it's continuous at every point in (a,b) and satisfies two additional conditions. For example, for the function f(x) = x^2 + 2x, which is continuous over its domain (-∞,-2) ∪ (-2,0) ∪ (0,+∞), we need to examine its behavior on either side of certain points. Additionally, the Composite Function Theorem allows us to expand our ability to compute limits and demonstrate that trigonometric functions are continuous over their domains. To evaluate the limit of the function cos(x-π/2), we apply the composite function theorem. Since the inner function x-π/2 approaches 0 as x approaches π/2, and cos(x) is continuous at 0, we can substitute this value into cos(x). This gives us a value of 1. Next, we'll show that trigonometric functions are continuous over their entire domains. To do this, we need to prove that cos(x) is continuous at every real number. We can use the composite function theorem again by showing that lim x→a cos(x) = cos(a) for all values of a. We then apply the identity for the cosine of the sum of two angles and simplify to get lim x→a cos(x) = cos(0) = 1. This shows that cos(x) is continuous at every real number, which means it's continuous over its entire domain. Using similar logic, we can show that sin(x) is also continuous over its entire domain. Since the other trigonometric functions can be expressed in terms of sin(x) and cos(x), their continuity follows from the quotient limit law. Finally, we're introduced to the Intermediate Value Theorem, which states that if a function is continuous over a closed interval, then it must take on every value between its maximum and minimum values at least once. We'll use this theorem to show that a specific function has at least one zero. The goal is to determine if the function f(x) = x - cos(x) has any zeros in the interval [0, π/2]. It is shown that this function satisfies the Intermediate Value Theorem (IVT), which states that since f(0) < 0 and f(π/2) > 0, there must be a real number c in [0, π/2] such that f(c) = 0. Therefore, we can conclude that f(x) has at least one zero. However, the IVT only allows us to find one value where the function changes sign, but it does not guarantee that there are no other zeros. To illustrate this, consider two examples: one where a continuous function has multiple zeros and another where a non-continuous function does not have any zeros in a given interval. The text also presents exercises related to determining discontinuities of various functions, including jump, removable, infinite, and other types of discontinuity. Additionally, it asks students to determine whether specific functions are continuous at certain points or values, and to find the value(s) of k that makes each function continuous over a given interval. Finally, the text includes exercises using the Intermediate Value Theorem (IVT), which can be used to find zeros of a function in a given interval. **Exercises** 151: A particle moves along a line with position function s(t). Another particle has position function h(t) = s(t) - t. Prove that there must be a value c such that 2 < c < 5, where h(c) = 0. 152: Write a mathematical equation representing the statement "The cosine of t is equal to t cubed." Use a calculator to find an interval containing a solution. 153: Apply IVT to determine if the equation x^3 - 2x has a solution in either [1.25, 1.375] or [1.375, 1.5]. 154: Identify all values where the function y = f(x) is discontinuous and explain why the formal definition of continuity doesn't apply. 155: Sketch the graph of f(x) = {3x, x > 1; x^3, x < 1}. Is it possible to make f(x) continuous for all real numbers? 156: Sketch the graph of f(x) = (x^4 - 1) / (x^2 - 1) for x ≠ -1 and 1. Can you find values k1 and k2 such that f(-1) = k1, f(1) = k2, and f(x) is continuous for all real numbers? 157: Sketch the graph of y = f(x) with properties: infinite discontinuity at x = -6, left-continuous but not right-continuous at x = 3. 158: Sketch the graph of y = f(x) with properties: removable discontinuity at x = 1, jump discontinuity at x = 2, and limits lim x→3−f(x) = -∞ and lim x→3+f(x) = 2. **Other Exercises** 159: Sketch a graph that is discontinuous at x = 1 with limits lim x→-1f(x) = -1 and lim x→2f(x) = 4. 160: Determine whether each of the given statements is true or false. Justify your response with an explanation or counterexample. 161: Is the function f(t) = 2et - e^(-t) continuous everywhere? 162: If a function has equal left- and right-hand limits as x approaches a, can it be discontinuous at that point? 163: Is each of these statements true or false? Justify your response with an explanation or counterexample. then the solution of cosx−sinx−x=2cosx−sinx−x=2 over the interval [−1,1] cannot be determined because it does not exist at that point. f(x) is continuous on [a,b] if f(a)f(b) have opposite signs. The function f(x)=x2−4x+3 has a discontinuity at x=1 and x=3. F(r)=ke|q1q2|r2 describes the electrostatic force between two point charges. It can be simplified by setting F as zero when r>R, where R is the threshold value, due to the physical reasoning that the electrostatic force becomes negligible for points very close to each other. The force equation using Coulomb's law and the approximation is F(r)=ke|q1q2|r2 and F(r)={−ke|q1q2|r2} if r≥R respectively. When R0, this system remains continuous. Letting the force be 10−20 for r≥R instead of zero also makes the system continuous. A possible value of R can make this system continuous as 10^(-7) < R < 1m The function F(d)=−mk/d2 describes the gravitational effect on a rocket. The constant k is given by k=m*d^2/2 where m is the mass of the rocket and d is the distance from Earth's center. Using the value of k, we can find the necessary condition D for which the force function remains continuous as √(2m*10^4) The function F(d)=−m1kd2ifd